# STRENGTH DESIGN OF CONCRETE MASONRY WALLS FOR AXIAL LOAD \& FLEXURE 

Structural

Keywords: axial strength, design aids, design example, interaction diagrams, loadbearing walls, load combinations, strength design, flexural strength, reinforced concrete masonry, structural design

## INTRODUCTION

The structural design of buildings requires a variety of loads to be accounted for: dead and live loads, those from wind, earthquake, lateral soil pressure, lateral fluid pressure as well as forces induced by temperature changes, creep, shrinkage and differential movements. Because most loads can act simultaneously with another, the designer must consider how these various loads interact on the wall. For example, a concentrically applied compressive axial load can offset tension due to lateral load, effectively increasing flexural capacity. Building codes dictate which load combinations must be considered, and require that the structure be designed to resist all possible combinations.

The design aids in this TEK cover combined axial compression or axial tension and flexure, as determined using the strength design provisions of Building Code Requirements for Masonry Structures (ref. 3). For concrete masonry walls, these design provisions are outlined in TEK 144A, Strength Design of Concrete Masonry (ref. 1). Axial load-bending moment interaction diagrams account for the interaction between moment and axial load on the design capacity of a wall. This TEK shows the portion of the interaction diagram that applies to the majority of wall designs. Although negative moments are not shown, the figures may be used for these conditions, since reinforcement in the center of the wall will provide equal strength under either a positive or negative moment of the same magnitude. Conditions outside of this area may be determined using Concrete Masonry Wall Design Software or Concrete Masonry Design Tables (refs. 4, 5). The reader is referred to Loadbearing Concrete Masonry Wall Design (ref. 2) for a full discussion of interaction diagrams.

Figures 1 through 8 apply to fully or partially grouted reinforced concrete masonry walls with a specified compressive strength $f_{m}^{\prime}$ of $1,500 \mathrm{psi}(10.34 \mathrm{MPa})$, and a maximum wall height of $20 \mathrm{ft}(6.10 \mathrm{~m})$, Grade $60(414 \mathrm{MPa})$ vertical reinforcement, with reinforcing bars positioned in the center of the wall
and reinforcing bar spacing $s$ from 8 in. to 120 in . ( 203 to 3,048 mm ). The following discussion applies to simply supported walls and is limited to uniform lateral loads. Other support and loading conditions should comply with applicable engineering procedures. Each figure applies to one specific wall thickness and one reinforcing bar size.

In strength design, two different deflections are calculated; one for service level loads $\left(\delta_{s}\right)$ and another for factored loads $\left(\delta_{u}\right)$. For a uniformly loaded simply supported wall , the resulting bending moment is as follows:
$M_{x}=W_{x} h^{2} / 8+P_{x f}(e / 2)+P_{x} \delta_{x}$
(Eqn. 1)
In the above equation, notations with " $x$ " are replaced with factored or service level values as appropriate. The first term on the right side of Equation 1 represents the maximum moment of a uniform load at the mid-height of the wall (normally wind or earthquake loads). The second term represents the moment induced by eccentrically applied floor or roof loads. The third term is the P-delta effect, which is the moment induced by vertical axial loads and lateral deflection of the wall.

## DESIGN EXAMPLE

An 8-in. (203-mm) thick, 20 ft ( 6.10 m ) high reinforced simply supported concrete masonry wall ( $115 \mathrm{pcf}\left(1,842 \mathrm{~kg} / \mathrm{m}^{3}\right)$ ) is to be designed to resist wind load as well as eccentrically applied axial live and dead loads as depicted in Figure 9. The designer must determine the reinforcement size spaced at 24 in. $(610 \mathrm{~mm})$ required to resist the applied loads, listed below.
$D=520 \mathrm{lb} / \mathrm{ft}(7.6 \mathrm{kN} / \mathrm{m})$, at $e=0.75 \mathrm{in} .(19 \mathrm{~mm})$
$L$
$=250 \mathrm{lb} / \mathrm{ft}(3.6 \mathrm{kN} / \mathrm{m})$, at $e=0.75 \mathrm{in} .(19 \mathrm{~mm})$
$W$

The wall weight at midheight for $115 \mathrm{pcf}\left(1,842 \mathrm{~kg} / \mathrm{m}^{3}\right)$ unit concrete density is $49 \mathrm{lb} / \mathrm{ft}^{2}\left(239 \mathrm{~kg} / \mathrm{m}^{2}\right)($ ref. 7 , Table 1$)$.

$$
\begin{aligned}
P_{w} \quad & =\left(49 \mathrm{lb} / \mathrm{ft}^{2}\right)(10 \mathrm{ft}) \\
& =490 \mathrm{lb} / \mathrm{ft}(7.2 \mathrm{kN} / \mathrm{m})
\end{aligned}
$$



Figure 1—8-Inch (203-mm) Concrete Masonry Wall With No. 4 (M\# 13) Reinforcing Bars


Figure 2-8-Inch (203-mm) Concrete Masonry Wall With No. 5 (M\#16) Reinforcing Bars


Figure 3-8-Inch (203-mm) Concrete Masonry Wall With No. 6 (M\# 19) Reinforcing Bars


Figure 4—10-Inch (254-mm) Concrete Masonry Wall With No. 4 (M\# 13) Reinforcing Bars


Figure 5—10-Inch (254-mm) Concrete Masonry Wall With No. 5 (M\# 16) Reinforcing Bars

The applicable load combination (ref. 6) for this example is: $1.2 D+1.6 W+f_{l} L+0.5 L_{r}$
(Eqn. 2)
During design, all load combinations should be checked. For brevity, only the combination above will be evaluated here.

First determine the cracking moment $M_{c r}$ :
$M_{c r}=S_{n} f_{r}=9,199 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}(3,410 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m})$, where
$S_{n}=93.2 \mathrm{in} .^{3} / \mathrm{ft}\left(5.01 \times 10^{6} \mathrm{~mm}^{3} / \mathrm{m}\right) \quad($ ref. 8, Table 1$)$
$f_{r}=98.7 \mathrm{psi}(0.68 \mathrm{MPa})$
(ref. 1, Table 1 interpolated for grout at 24 in . $(610 \mathrm{~mm})$ o.c.)
To check service level load deflection and moment, the following analysis is performed in an iterative process.
First iteration, $\delta_{\mathrm{s}}=0$
$M_{\text {serl }}=20(20)^{2}(12) / 8+(520+250)(0.75 / 2)+(520+250+490)(0)$ $=12,289 \mathrm{in} .-\mathrm{lb} / \mathrm{ft}(4,555 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m})$ (fromEqn. 1)
Since $M_{c r}<M_{\text {ser } 1}$, therefore analyze as a cracked section.
$\delta_{s 1}=\frac{5 M_{c r} h^{2}}{48 E_{m} I_{g}}+\frac{5\left(M_{s e r}-M_{c r}\right) h^{2}}{48 E_{m} I_{c r}}(12)$
(Eqn. 3)
where: $E_{m}=900 f_{m}^{\prime}=1,350,000 \mathrm{psi}(9,308 \mathrm{MPa})$
$I_{g}=369.4 \mathrm{in} .4 / \mathrm{ft}\left(504 \times 10^{6} \mathrm{~mm}^{4} / \mathrm{m}\right) \quad$ (ref.8,Table 1)
$I_{c r}^{g}=21.0 \mathrm{in} .^{4} / \mathrm{ft}\left(504 \times 10^{6} \mathrm{~mm}^{4} / \mathrm{m}\right) \quad$ (Table 1)

$$
\begin{aligned}
\delta_{s 1} & =\frac{5(9,199)(240)^{2}}{48(1,350,000)(369.4)}+\frac{5(12,289-9,199)(240)^{2}}{48(1,350,000)(21.0)} \\
& =0.76 \mathrm{in} .(19 \mathrm{~mm})
\end{aligned}
$$

Second iteration, $\delta_{\mathrm{s}}=0.76 \mathrm{in}$. $(19 \mathrm{~mm})$
$M_{s e r 2}=12,289+(520+250+490)(0.76)$
$=13,247 \mathrm{in} .-\mathrm{lb} / \mathrm{ft}(4,910 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m})$
$\delta_{\mathrm{s} 2}=0.97 \mathrm{in} .(25 \mathrm{~mm})$
Thirditeration, $M_{\text {ser } 2}=13,511 \mathrm{in} .-\mathrm{lb} / \mathrm{ft}(5,008 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m}), \delta_{\mathrm{s} 3}=1.02$ in. $(26 \mathrm{~mm})$. Because $\delta_{\mathrm{s} 3}$ is within $5 \%$ of $\delta_{\mathrm{s} 2}$, then $\delta_{\mathrm{s}}=\delta_{\mathrm{s} 3}$.

Check $\delta_{\mathrm{s}}$ against the maximum service load deflection: $\delta_{\text {s }}$ $<0.007 \mathrm{~h}=0.007(240)=1.68 \mathrm{in}$. $(43 \mathrm{~mm})>1.02 \mathrm{in} .(26 \mathrm{~mm})$, OK.

If $M_{s e r}<M_{c r}$, instead of using Equation 2 for deflection, we would have used:


Figure 6-12-Inch (305-mm) Concrete Masonry Wall With No. 4 (M\# 13) Reinforcing Bars


Figure 7—12-Inch (305-mm) Concrete Masonry Wall With No. 5 (M\#16) Reinforcing Bars


Figure 8-12-Inch (305-mm) Concrete Masonry Wall With No. 6 (M\# 19) Reinforcing Bars

| Table 1-Cracked Moment of Inertia, $I_{c r}$, in. $/ / \mathrm{ft}^{\text {a }}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Barsize, <br> No. (M \#) | Spacing of reinforcement, in. (mm) |  |  |  |  |  |  |  |  |
|  | 8(203) | 16(406) | 24(610) | 32(813) | 40(1,016) | 48(1,219) | 72(1,829) | 96(2,438) | $120(3,048)$ |
| 8 -inch (203-mm) wall thickness: |  |  |  |  |  |  |  |  |  |
| 4 (13) | 47.9 | 28.9 | 21.0 | 16.6 | 13.7 | 11.8 | 8.25 | 6.38 | 5.21 |
| 5 (16) | 63.8 | 40.0 | 29.6 | 23.7 | 19.8 | 17.0 | 12.1 | 9.42 | 7.74 |
| 6 (19) | 78.5 | 51.0 | 38.5 | 31.1 | 26.2 | 22.7 | 16.3 | 12.8 | 10.5 |
| 10-inch (254-mm) wall thickness: |  |  |  |  |  |  |  |  |  |
| 4 (13) | 81.8 | 48.5 | 34.9 | 27.4 | 22.6 | 19.3 | 13.5 | 10.4 | 8.47 |
| 5 (16) | 110.5 | 67.9 | 49.7 | 39.5 | 32.9 | 28.2 | 19.9 | 15.4 | 12.6 |
| 12-inch (305-mm) wall thickness: |  |  |  |  |  |  |  |  |  |
| 4 (13) | 125.7 | 73.4 | 52.5 | 41.1 | 33.8 | 28.8 | 20.0 | 15.4 | 12.5 |
| 5 (16) | 171.6 | 103.7 | 75.4 | 59.6 | 49.4 | 42.3 | 29.7 | 23.0 | 18.8 |
| 6 (19) | 216.1 | 134.3 | 99.4 | 79.3 | 66.2 | 56.9 | 40.3 | 31.4 | 25.7 |
| ${ }^{\text {a }}$ Intermediate spacings may be interpolated. |  |  |  |  |  |  |  |  |  |

$\delta_{s}=\frac{5 M_{s e r} h^{2}}{48 E_{m} I_{g}}$
To determine deflection and moment due to factored loads, an identical calculation is performed as for service loads with the exception that factored loads are used in Equations 1 and 3 or Equations 1 and 4 .

First iteration, $\delta_{\mathrm{u}}=0$, using Equation 1 :
lateral $=1.6\left[(20)(20)^{2}(12) / 8\right]=19,200$
roof \& floor $=1.2(520)(0.75 / 2)+0.5(250)(0.75 / 2)=281$
P-delta $=[1.2(520+490)+0.5(250)] 0=0$
$M_{u 1}=$ lateral+roof \& floor+P-delta= $19,481 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}(7,221 \mathrm{mN} / \mathrm{m})$
From Equation 3, using $M_{u 1}$ instead of $M_{s e r}, \delta_{u 1}=2.29$ in. (58 mm ).
Second iteration, $M_{u 2}=22,543 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}(8,356 \mathrm{mN} / \mathrm{m}), \delta_{\mathrm{u} 2}=2.94$ in. $(75 \mathrm{~mm})$.
Third iteration, $M_{u 3}=23,412 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}(8,678 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m}), \delta_{\mathrm{u} 3}=3.12$ in. $(79 \mathrm{~mm})$.
Fourth iteration, $M_{u 4}=23,652 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}(8,767 \mathrm{~m} \cdot \mathrm{~N} / \mathrm{m}), \delta_{\mathrm{u} 4}=3.17$ in. $(81 \mathrm{~mm})$.
$\delta_{\mathrm{u} 4}$ is within $5 \%$ of $\delta_{\mathrm{u} 3}$. Therefore, $M_{u}=M_{u 4}=23,652 \mathrm{lb}-\mathrm{in} . / \mathrm{ft}=$ $1,971 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}(8,767 \mathrm{~m} \mathrm{~N} / \mathrm{m})$.

$$
\begin{aligned}
P_{u} & =1.2(520+490)+0.5(250) \\
& =1,337 \mathrm{lb} / \mathrm{ft}(20 \mathrm{kN} / \mathrm{m})
\end{aligned}
$$

To determine the required reinforcement size and spacing to resist these loads, $P_{u}$ and $M_{u}$ are plotted on the appropriate interaction diagram until a satisfactory design is found. If the axial load is used to offset stresses due to bending, only the unfactored dead load should be considered

Figure 1 shows that No. 4 bars at 24 in . (M\#13 at 610 mm ) on center is adequate. If a larger bar spacing is desired, No. 5 at 32 in . (M\#16 at 813 mm ) orNo. 6 at 48 in . (M\#19 at 1219 mm ) also appear to meet the design requirements (see Figures 2 and 3 , respectively). However, the design procedure should be
repeated and verified with the new grout spacings and associated properties. Although above grade wall design is seldom governed by out-of-plane shear, the shear capacity should be checked.

## NOMENCLATURE

$D$ dead load, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} / \mathrm{m})$
$E_{m} \quad$ modulus of elasticity of masonry in compression, psi (MPa)
$e \quad$ eccentricity of axial load-measured from centroid of wall, in. (mm)
$f_{m}^{\prime} \quad$ specified masonry compressive strength, psi (MPa)
$f_{r}^{m} \quad$ modulus of rupture, psi (MPa)
$f_{1} \quad$ factor for floor load: $=1.0$ for floors in places of public assembly, for live loads in excess of $100 \mathrm{psf}(4.8 \mathrm{kPa})$ and for parking garage live loads; $=0.5$ otherwise
$h$ height of wall, in. (mm)
$I_{c r} \quad$ moment of inertia of cracked cross-sectional area of a member, in. $/$ /ft $\left(\mathrm{mm}^{4} / \mathrm{m}\right)$
$I_{g}$ moment of inertia of gross cross-sectional area of a member, taken here as equal to $I_{\text {avg }}$, in. ${ }^{4} / \mathrm{ft}\left(\mathrm{mm}^{4} / \mathrm{m}\right)$
$L \quad$ live load, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} / \mathrm{m})$
$L_{r} \quad$ roof live load, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} / \mathrm{m})$
$M_{c r}$ nominal cracking moment strength, in. $-\mathrm{lb} / \mathrm{ft}(\mathrm{kN} \cdot \mathrm{m} / \mathrm{m})$
$M_{s e r}$ service moment at midheight of a member, including Pdelta effects, in.-lb/ft (kN•m/m)
$M_{u} \quad$ factored moment, in.- $\mathrm{lb} / \mathrm{ft}$ or $\mathrm{ft}-\mathrm{lb} / \mathrm{ft}(\mathrm{kN} \cdot \mathrm{m} / \mathrm{m})$
$P_{u} \quad$ factored axial load, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} / \mathrm{m})$
$P_{u f}^{u}$ factored load from tributary floor or roof areas, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} /$ m)
$P_{w} \quad$ load due to wall weight, $\mathrm{lb} / \mathrm{ft}(\mathrm{kN} / \mathrm{m})$
$S_{n}^{w} \quad$ section modulus of the net cross-sectional area of a member, in. ${ }^{3} / \mathrm{ft}\left(\mathrm{mm}^{3} / \mathrm{m}\right)$
$s \quad$ spacing of vertical reinforcement, in. (mm)
$W$ wind load, $\mathrm{psf}\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
$\delta_{\mathrm{s}}$ horizontal deflection at midheight under service loads, in. (mm)
$\delta_{u}$ deflection due to factored loads, in. (mm)


## REFERENCES

1. Strength Design of Concrete Masonry, TEK 14-4A. National Concrete Masonry Association, 2002.
2. Loadbearing Concrete Masonry Wall Design, TEK 14-5A. National Concrete Masonry Association, 2000.
3. Building Code Requirements for Masonry Structures, ACI 530-02/ASCE 5-02/TMS 402-02. Reported by the Masonry Structures Joint Committee, 2002.
4. Concrete Masonry Wall Design Software, CMS-10. National Concrete Masonry Association, 2002.
5. Concrete Masonry Design Tables, TR 121A. National Concrete Masonry Association, 2000.
6. Minimum Design Loads for Buildings and Other Structures, ASCE 7-02. American Society of Civil Engineers, 2002.
7. Concrete Masonry Wall Weights, TEK 14-13A. National Concrete Masonry Association, 2002.
8. Section Properties of Concrete Masonry Walls, TEK 14-1. National Concrete Masonry Association, 1993.

## METRICCONVERSIONS

| To convert: | To metric units: | Multiply English units by: |
| :---: | :---: | :---: |
| ft | m | 0.3048 |
| $\mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | $\mathrm{mN} / \mathrm{m}$ | 4.44822 |
| $\mathrm{lb}-\mathrm{in} / \mathrm{ft}$ | $\mathrm{mN} / \mathrm{m}$ | 0.37069 |
| in. | mm | 25.4 |
| $\mathrm{in} .4 / \mathrm{ft}$ | $\mathrm{mm} / \mathrm{m}$ | $1,366,000$ |
| $\mathrm{lb} / \mathrm{ft}$ | $\mathrm{kN} / \mathrm{m}$ | 0.0145939 |
| psi | MPa | 0.00689476 |

