NCMA TEK

National Concrete Masonry Association
an information series from the national authority on concrete masonry technology

ALLOWABLE STRESS DESIGN OF CONCRETE MASONRY

TEK 14-7A
Structural (2004)

Keywords: allowable loads, allowable stress, allowable stress design, axial strength, building code provisions, flexural strength, reinforced concrete masonry, shear strength, structural design, unreinforced concrete masonry

INTRODUCTION

Loadbearing concrete masonry walls can be designed using one of several methods in accordance with *Building Code Requirements for Masonry Structures* (ref. 1): empirical design, strength design or allowable stress design (ASD). This TEK provides a basic overview of design criteria and requirements for concrete masonry walls designed using the ASD provisions contained in Chapter 2 of *Building Code Requirements for Masonry Structures*. For the empirical or strength design provisions, the reader is referred to TEK 14-8A *Empirical Design of Concrete Masonry Walls* and TEK 14-4A *Strength Design of Concrete Masonry* (refs. 2, 3), respectively.

This TEK is intended only to provide a general review of the pertinent ASD criteria. Tables, charts and additional aids specific to the allowable stress design of concrete masonry elements can be found in other related TEK.

Allowable stress design is based on the following design principles and assumptions.

- Within the range of allowable stresses, masonry elements satisfy applicable conditions of equilibrium and compatibility of strains.
- Plane sections before bending remain plane after bending. Therefore, masonry strain is directly proportional to the distance from the neutral axis.
- Stress is linearly proportional to strain within the allowable stress range.
- For reinforced masonry design, all tensile stresses are resisted by the steel reinforcement. The contribution of the masonry to the tensile strength of the element is ignored.
- The units, mortar, grout and reinforcement, if present, act compositely to resist applied loads.

Based on this assumed design model, the internal distribution of stresses and resulting equilibrium is illustrated in Figure 1 for unreinforced masonry and Figure 3 for reinforced masonry.

DESIGN LOADS

Utilizing allowable stress design, masonry elements are sized and proportioned such that the anticipated service level loads can be safely and economically resisted using the specified material strengths. The specified strength of masonry and reinforcement are in turn reduced by appropriate safety factors. Minimum design loads for allowable stress design are included in *Minimum Design Loads for Buildings and Other Structures* (ref. 4) or obtained from the general building code such as the *International Building Code* (IBC) (ref. 5). For load combinations that include wind or earthquake loads, the code-prescribed allowable stresses (refs. 1, 5) are permitted to be increased by one-third. Note, however, that the one-third stress increase cannot be used when using the loads or load combinations of *Minimum Design Loads for Buildings and Other Structures* (ref. 4).

Using ASD, the calculated design stresses on a masonry member (indicated by lowercase f) are compared to code-prescribed maximum allowable stresses (indicated by a capital F). The design is acceptable when the calculated applied stresses are less than or equal to the allowable stresses ($f < F$).

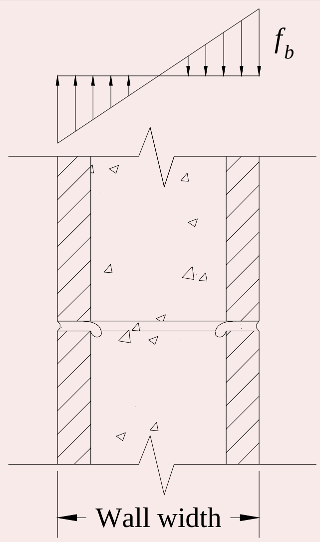


Figure 1—Unreinforced Masonry Stress Distribution

UNREINFORCED MASONRY

For unreinforced masonry, the masonry assembly (units, mortar and grout if used) is designed to carry all applied stresses. The additional capacity from the inclusion of reinforcing steel, such as reinforcement added for the control of shrinkage cracking or prescriptively required by the code, is neglected. Because the masonry is intended to resist both tension and compression stresses resulting from applied loads, the masonry must be designed to remain uncracked.

Unreinforced Out-of-Plane Flexure

Allowable flexural tension values as prescribed in *Building Code Requirements for Masonry Structures* (ref. 1), vary with the direction of span, mortar type, bond pattern and percentage of grouting as shown in Table 1. For walls spanning horizontally between supports, the code conservatively assumes that masonry constructed in stack bond cannot reliably transfer flexural tension stresses across the head joints. As such, the allowable flexural tension values parallel to the bed joints (perpendicular to the head joints) for stack bond construction are assumed to be zero for design purposes.

Because the compressive strength of masonry is much larger than its tensile strength, the capacity of unreinforced masonry subjected to net flexural stresses is almost always controlled by the flexural tension values of Table 1. For masonry elements subjected to a bending moment, *M*, and a compressive axial force, *P*, the resulting flexural bending stress is determined using Equation 1.

Table 1—Allowable Flexural Tension Stresses for Unreinforced Concrete Masonry, psi (kPa) (ref. 1)

Direction of flexural tensile stress & masonry type	Mortar types: Portland cement/lime or mortar cement, M or S	Portland cement/lime or mortar cement, N	Masonry cement or air entrained portland cement/lime, M or S	Masonry cement or air entrained portland cement/lime, N
Normal to bed joints (walls spanning vertically):				
Solid units	40 (276)	30 (207)	24 (166)	15 (103)
Hollow units[a]				
Ungrouted	25 (172)	19 (131)	15 (103)	9 (62.1)
Fully grouted	65 (448)	63 (434)	61 (420)	58 (400)
Parallel to bed joints in running bond (walls spanning horizontally):				
Solid units	80 (552)	60 (414)	48 (331)	30 (207)
Hollow units				
Ungrouted & partially grouted	50 (345)	38 (262)	30 (207)	19 (131)
Fully grouted	80 (552)	60 (414)	48 (331)	30 (207)

[a] For partially grouted masonry, allowable stresses are determined by linear interpolation between fully grouted hollow units and ungrouted hollow units based on the amount of grouting.

$$f_b = \frac{Mt}{2I_n} - \frac{P}{A_n} \qquad \text{Eqn. 1}$$

TEK 14-1 *Section Properties of Concrete Masonry Walls* (ref. 6) provides typical values for the net moment of inertia, I_n, and cross-sectional area, A_n, for various wall sections. If the value of the bending stress, f_b, given by Equation 1 is positive, then the masonry section is controlled by tension and the limiting values of Table 1 must be satisfied. Conversely, if f_b as given by Equation 1 is negative, the masonry section is in compression and the compressive stress limitation of Equation 2 must be met.

$$f_b \leq F_b = \tfrac{1}{3} f'_m \qquad \text{Eqn. 2}$$

Unreinforced Axial Compression and Flexure

While unreinforced masonry can be designed to resist flexural tension stresses due to applied loads, unreinforced masonry may not be subjected to net axial tension, such as that due to wind uplift on a roof connected to a masonry wall or due to the overturning effects of lateral loads. While compressive stresses from dead loads can be used to offset tensile stresses, where the wall is subject to a net axial tension reinforcement must be incorporated to resist the resulting tensile forces.

When masonry walls are subjected to compressive axial loads only, the calculated compressive stress due to the applied load, f_a, must not exceed the allowable compressive stress, F_a, as given by Equation 3 or 4, as appropriate.
For elements with *h*/*r* not greater than 99:

$$f_a \leq F_a = \tfrac{1}{4} f'_m \left[1 - \left(\frac{h}{140r}\right)^2\right] \qquad \text{Eqn. 3}$$

For elements with *h*/*r* greater than 99:

$$f_a \leq F_a = \tfrac{1}{4} f'_m \left(\frac{70r}{h}\right)^2 \qquad \text{Eqn. 4}$$

A further check for stability against an eccentrically applied axial load is included with Equation 5, whereby the axial compressive load, *P*, is limited to one-fourth the buckling load, P_e. With Equation 5, the actual eccentricity of the applied load, *e*, is used to determine P_e. Moments on the wall due to loads other than the eccentric load are not considered in Equation 5.

$$P \leq \tfrac{1}{4} P_e = \left(\tfrac{1}{4}\right) \frac{\pi^2 E_m I_n}{h^2} \left(1 - 0.577 \, e/r\right)^3 \qquad \text{Eqn. 5}$$

When unreinforced masonry elements are subjected to a combination of axial load and flexural bending, a unity equation is used to proportion the available allowable stresses to the applied loads per Equation 6.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1 \qquad \text{Eqn. 6}$$

Unreinforced Shear

Shear stresses on unreinforced masonry elements are calculated based on the net cross-sectional properties of the masonry in the direction of the applied shear force using the following relation:

$$f_v = \frac{VQ}{I_n b} \qquad \text{Eqn. 7}$$

Equation 7 is applicable to the determination of both in-plane and out-of-plane shear stresses. Because unreinforced masonry is designed to remain uncracked, it is not necessary to perform a cracked section analysis to determine the net cross-sectional area of the masonry.

The theoretical distribution of shear stress, f_v, along the length of the shear wall for in-plane loads (Figure 2), or perpendicular to any wall for out-of-plane loads, has a parabolic shape for a rectangular cross-section. The calculated shear stress due to applied loads, f_v, as given by Equation 7 is limited to the code-prescribed allowable shear stress, F_v, from Table 2. While *Building Code Requirements for Masonry Structures* (ref. 1) designates the values in Table 2 as being applicable to in-plane shear stresses only, no allowable shear stresses are provided for out-of-plane loads. In light of the absence of out-of-plane allowable shear stress values, the *Commentary on Building Code Requirements for Masonry Structures* suggests using the values in Table 2 for out-of-plane shear design.

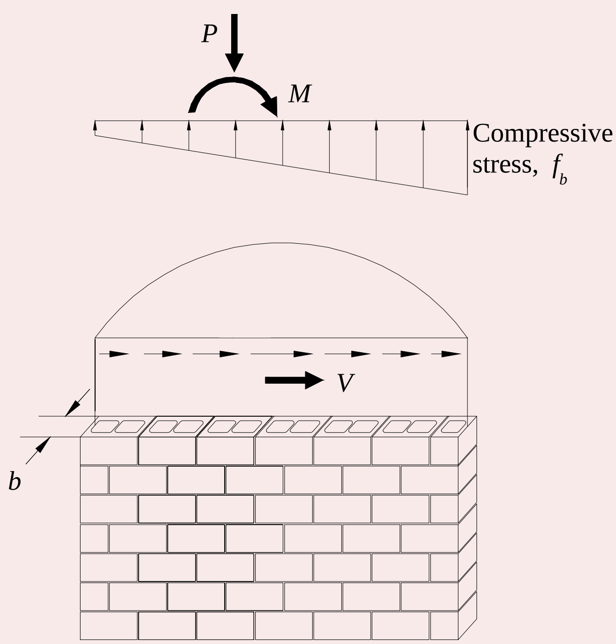


Figure 2—Unreinforced Masonry Shear Walls

Table 2—Allowable Shear Stresses for Unreinforced Masonry

Allowable shear stress, F_v, is the least of:

1. $1.5\sqrt{f'_m}$ psi $\quad (0.125\sqrt{f'_m}$ MPa)
2. 120 psi $\quad$ (827 kPa)
3. a. For running bond not solidly grouted and for stack bond masonry with open end units and grouted solid, 37 psi + $0.45N_v/A_n$ $\quad$ (255 kPa + $0.45N_v/A_n$)
   b. For masonry in running bond and solidly grouted, 60 psi + $0.45N_v/A_n$ $\quad$ (414 kPa + $0.45N_v/A_n$)
   c. For stack bond masonry with other than open end units grouted solid, 15 psi (103 kPa)

REINFORCED MASONRY

The design of reinforced masonry in accordance with *Building Code Requirements for Masonry Structures* (ref. 1) neglects the tensile resistance provided by the masonry units, mortar and grout in determining the strength of the masonry assemblage. Thus, for design purposes, the portion of masonry subjected to net tensile stresses is assumed to have cracked, transferring all tensile forces to the reinforcement. (While the determination of the capacity of a reinforced masonry element conservatively assumes the portion of the masonry subjected to net tensile stresses has cracked, this should be verified when establishing the stiffness and deflection of a reinforced masonry element.)

Reinforcement

The tensile stress in the reinforcement due to applied load, f_s, is calculated as the product of the strain in the steel (which increases linearly in proportion to the distance from the neutral axis) multiplied by its modulus of elasticity, E_s. The modulus of elasticity, E_s, of mild steel reinforcement is assumed to be 29,000,000 psi (200 GPa). The code-prescribed allowable steel stresses are as follows (ref. 1).

For Grade 60 reinforcement in tension:
F_s = 24,000 psi (165.5 MPa)
For Grade 40 and 50 reinforcement in tension:
F_s = 20,000 psi (137.9 MPa)
For wire reinforcement in tension:
F_s = 30,000 psi (206.9 MPa)
For all reinforcement in compression:
F_s = 24,000 psi (165.5 MPa) or $0.4f_y$, whichever is less

Unless ties or stirrups laterally confine the reinforcement as prescribed by *Building Code Requirements for Masonry Structures*, the reinforcement is assumed to contribute no compressive resistance to axially loaded elements. Additional information on mild reinforcing steel can be found in TEK 12-4C *Steel Reinforcement for Concrete Masonry* (ref. 7).

For design purposes, the effective width of the compression zone per bar is limited to the smallest of:

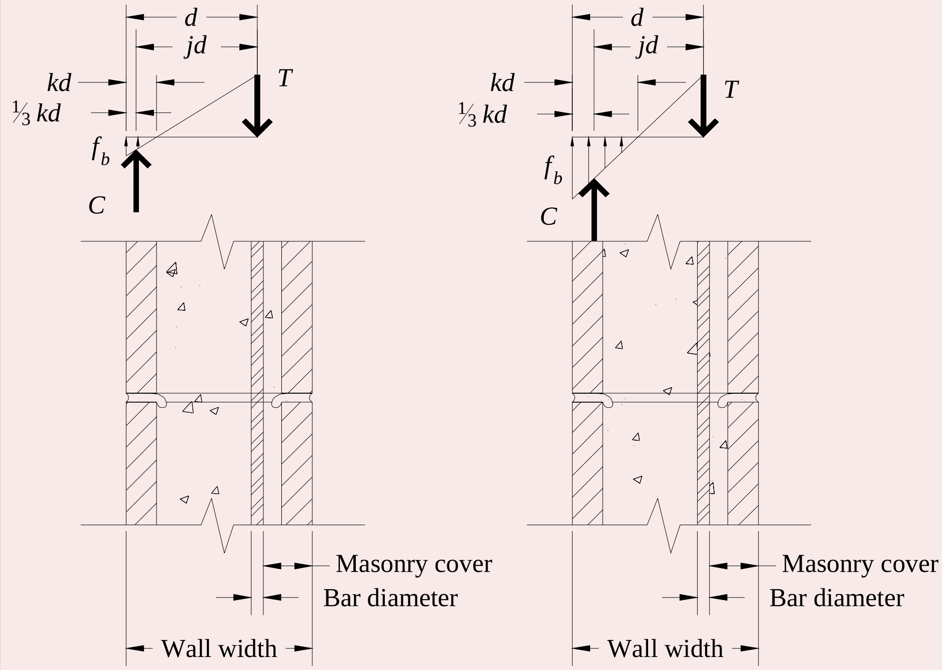
- six times the wall thickness,
- the center-to-center spacing of the reinforcement, or
- 72 in. (1,829 mm).

This requirement applies to masonry constructed in running bond and to masonry constructed in stack bond containing bond beams spaced no further than 48 in. (1,219 mm) on center. Where the center-to-center spacing of the reinforcement does not control the effective width of the compression zone, the resulting resisting moment or resisting shear is proportioned over the width corresponding to the actual reinforcement spacing.

Reinforced Out-of-Plane Flexure

As with unreinforced masonry, the allowable compressive stress in masonry, F_b, due to flexure or due to a combination of flexure and axial load is limited by Equation 2. When axial loads are not present, or are conservatively neglected as may be appropriate in some cases, there are several circumstances to consider in determining the flexural capacity of reinforced masonry walls. For a fully grouted element, a cracked transformed section approach is used wherein the reinforcement area is transformed to an equivalent area of concrete masonry using the modular ratio. Partially grouted walls are analyzed in the same way, but with the additional consideration of the ungrouted cores. For partially grouted masonry there are two types of behavior to consider.

1. The first case applies when the neutral axis (the location of zero stress) lies within the compression face shell as shown in Figure 3a. In this case, the wall is analyzed and designed using the procedures for a fully grouted wall.
2. The second type of analysis occurs when the neutral axis lies within the core area, rather than the compression face shell, as shown in Figure 3b. For this case, the portion of the ungrouted cells must be deducted from the area of masonry capable of carrying compression stresses.



a—Neutral axis within the face shell **b—Neutral axis in core area**

Figure 3—Reinforced Masonry Stress Distribution

The location of the neutral axis depends on the relative moduli of elasticity of the masonry and steel, η, as well as the reinforcement ratio, ρ, and the distance between the reinforcement and the extreme compression fiber, d.

When analyzing partially grouted walls, it is typically assumed that the neutral axis lies within the compression face shell, as the analysis is more straightforward. Based on this assumption, the resulting value of k and the location of the neutral axis (kd) is calculated. If it is determined that the neutral axis lies outside the compression face shell, the more rigorous tee beam analysis is performed. Otherwise, the rectangular beam analysis is carried out. A complete discussion and derivation of this procedure is contained in *Concrete Masonry Design Tables* (ref. 8).

Rectangular Beam Analysis

For fully grouted masonry elements and for partially grouted masonry walls with the neutral axis in the compression face shell, the resisting flexural capacity, M_r, is calculated as follows:

$$\eta = E_s / E_m \qquad \text{Eqn. 8}$$

$$\rho = A_s / bd \qquad \text{Eqn. 9}$$

$$k = \sqrt{2\rho\eta + (\rho\eta)^2} - \rho\eta \qquad \text{Eqn. 10}$$

$$j = 1 - k/3 \qquad \text{Eqn. 11}$$

$$M_m = \tfrac{1}{2} F_b kjbd^2 \qquad \text{Eqn. 12}$$

$$M_s = A_s F_s jd \qquad \text{Eqn. 13}$$

The resisting flexural capacity, M_r, is taken as the lesser of M_m and M_s.

Tee Beam Analysis

For partially grouted masonry walls where the neutral axis is located within the cores, the resisting flexural capacity, M_r, is calculated using the neutral axis coefficient k given by Equation 14 and either Case A or Case B as follows:

$$k = \frac{-A_s\eta - t_{fs}(b - b_w)}{db_w} \qquad \text{Eqn. 14}$$

(A) For cases where the masonry strength controls the design capacity:

$$f_s = \eta F_b\left(\frac{1-k}{k}\right) \qquad \text{Eqn. 15}$$

If f_s as determined using Equation 15 is greater than the allowable steel stress, F_s, then the steel controls the design capacity per (B) below. Otherwise, the internal compression force, C, and tension force, T, are computed as follows:

$$C = \tfrac{1}{2} F_b bkd \qquad \text{Eqn. 16}$$

$$T = A_s f_s = A_s \eta F_b \left(1 - k \big/ k\right) \qquad \text{Eqn. 17}$$

(B) For cases where the steel strength controls the design:

$$f_b = \frac{F_s}{\eta}\left(\frac{k}{1-k}\right) \qquad \text{Eqn. 18}$$

$$C = \tfrac{1}{2} f_b bkd = \tfrac{1}{2} bkd\left(\frac{F_s}{\eta}\left(\frac{k}{1-k}\right)\right) \qquad \text{Eqn. 19}$$

$$T = A_s F_s \qquad \text{Eqn. 20}$$

(C) The design capacity is then computed using the following relationship:

$$M_r = C\left(\frac{2t_{fs}}{3}\right) - T\left(d - \frac{kd}{3} - \frac{2t_{fs}}{3}\right) \qquad \text{Eqn. 21}$$

$$V_r = F_v bd \qquad \text{Eqn. 22}$$

Reinforced Axial Compression

Axial loads acting through the axis of a member are distributed over the net cross-sectional area of masonry. The allowable axial compressive force is based on the compressive strength of masonry and the slenderness ratio of the element in accordance with the following.

For elements with h/r not greater than 99, the allowable compressive force, P_a, is:

$$P_a = \left(0.25 f'_m A_n + 0.65 A_s F_s\right)\left[1 - \left(\frac{h}{140r}\right)^2\right] \qquad \text{Eqn. 23}$$

For elements with h/r greater than 99, the allowable compressive force, P_a, is:

$$P_a = \left(0.25 f'_m A_n + 0.65 A_s F_s\right)\left(\frac{70r}{h}\right)^2 \qquad \text{Eqn. 24}$$

Reinforced Axial Compression and Flexure

Often, loading conditions result in both axial load and flexure on a wall. Superimposing the stresses resulting from axial compression and flexural compression produces the combined stress. Members are proportioned so that this maximum combined stress does not exceed the allowable stress limitation imposed by Equation 2, Equation 5, Equation 6 and either Equation 23 or 24, as appropriate. Several design approaches are available for combined axial compression and flexure, most commonly using either computer programs to perform the necessary iterative calculations or using interaction diagrams to graphically determine the required reinforcement for a given condition (refs. 8, 9, 10).

Reinforced Shear

Shear acting on masonry flexural members and shear walls is resisted either by the masonry (units, mortar and grout) or by shear reinforcement. For masonry members that are not subjected to flexural tension, the allowable shear stresses provided in Table 2 for unreinforced masonry apply. For masonry elements that are subjected to flexural tension, the applied shear stress is calculated as follows:

$$f_v = \frac{V}{bd} \qquad \text{Eqn. 25}$$

Where reinforcement is not provided to resist the entire calculated shear stress, f_v, the allowable shear stress, F_v, is determined in accordance with the following:

For flexural members:

$$F_v = \sqrt{f'_m} \le 50 \text{ psi} \qquad \text{Eqn. 26}$$

(metric: $F_v = 0.083\sqrt{f'_m} \le 345$ kPa)

For shear walls:

where $M/Vd < 1$,

$$F_v = \tfrac{1}{3}\left[4 - (M/Vd)\right]\sqrt{f'_m} \le 80 - 45(M/Vd) \qquad \text{Eqn. 27}$$

(metric: $F_v = 0.028\left[4 - (M/Vd)\right]\sqrt{f'_m} \le 0.55 - 0.31(M/Vd)$)

where $M/Vd \ge 1$,

$$F_v = \sqrt{f'_m} \le 35 \text{ psi} \qquad \text{Eqn. 28}$$

(metric: $F_v = 0.083\sqrt{f'_m} \le 241$ kPa)

When shear reinforcement is provided to resist the entire shear force, the minimum amount of shear reinforcement is determined by Equation 29.

$$A_v = \frac{Vs}{F_s d} \qquad \text{Eqn. 29}$$

Where reinforcement is provided to resist the entire calculated shear stress, f_v, the allowable shear stress, F_v, is determined in accordance with the following:

For flexural members:

$$F_v = 3\sqrt{f'_m} \le 150 \text{ psi (1034 kPa)} \qquad \text{Eqn. 30}$$

(metric: $F_v = 0.25\sqrt{f'_m} \le 1{,}034$ kPa)

For shear walls:

where $M/Vd < 1$,

$$F_v = \tfrac{1}{2}\left[4 - (M/Vd)\right]\sqrt{f'_m} \le 120 - 45(M/Vd) \qquad \text{Eqn. 31}$$

(metric: $F_v = 0.042\left[4 - (M/Vd)\right]\sqrt{f'_m} \le 0.82 - 0.31(M/Vd)$)

where $M/Vd \ge 1$,

$$F_v = 1.5\sqrt{f'_m} \le 75 \text{ psi} \qquad \text{Eqn. 32}$$

(metric: $F_v = 0.125\sqrt{f'_m} \le 517$ kPa)

For Equations 27, 28, 31, and 32, the ratio of M/Vd is required to be taken as a positive value.

Shear reinforcement provided in accordance with Equations 29 through 32 must also comply with the following.

- Shear reinforcement is oriented parallel to the direction of the shear force.
- Shear reinforcement spacing must not exceed the lesser of $d/2$ or 48 in. (1,219 mm).
- Reinforcement must also be provided perpendicular to the shear reinforcement. This prescriptive reinforcement must have an area of at least one-third A_v as given by Equation 28 and may not be spaced farther apart than 8 ft (2,438 mm).

NOTATION:

A_n = net cross-sectional area of masonry, in.^{2} (mm^2)
A_s = effective cross-sectional area of reinforcement, in.^{2} (mm^2)
A_v = cross-sectional area of shear reinforcement, in.^{2} (mm^2)
b = width of section, in. (mm)
b_w = for partially grouted walls, width of grouted cell plus each web thickness within the compression zone, in. (mm)
C = resultant compressive force, lb (N)

d = distance from the extreme compression fiber to centroid of tension reinforcement, in. (mm)
E_m = modulus of elasticity of masonry, psi (MPa)
E_s = modulus of elasticity of reinforcement, psi (MPa)
e = eccentricity of applied load, in. (mm)
F_a = allowable compressive stress due to axial load only, psi (MPa)
F_b = allowable bending stress due to flexure only, psi (MPa)
F_s = allowable tensile or compressive stress in reinforcement, psi (MPa)
F_v = allowable shear stress, psi (MPa)
f_a = calculated compressive stress due to axial load only, psi (MPa)
f_b = calculated bending stress due to flexure only, psi (MPa)
f'_m = specified compressive strength of masonry, psi (MPa)
f_s = calculated tensile or compressive stress in reinforcement, psi (MPa)
f_v = calculated shear stress in masonry, psi (MPa)
f_y = specified yield strength of reinforcement, psi (MPa)
h = effective height of masonry element, in. (mm)
I_n = moment of inertia of net cross-sectional area of a masonry element, in.^{4} (mm^4)
j = ratio of distance between centroid of flexural compressive forces and centroid of tensile forces to depth d
k = ratio of distance between compression face of wall and neutral axis to the effective depth d
M = maximum calculated bending moment at the section under consideration, in.-lb (N-mm)
M_m = flexural capacity (resisting moment) when masonry controls, in.-lb/ft (N-m/m)
M_r = flexural capacity (resisting moment), in.-lb/ft (N-m/m)
M_s = flexural capacity (resisting moment) when reinforcement controls, in.-lb/ft (N-m/m)
N_v = compressive force acting normal to the shear surface, lb (N)
P = applied axial load, lb (N)
P_a = allowable compressive force in reinforced masonry due to axial load, lb (N)
P_e = Euler buckling load, lb (N)
Q = first moment of inertia about the neutral axis, in.^{3} (mm^3)
r = radius of gyration, in. (mm)
s = spacing of shear reinforcement, in. (mm)
T = resultant tensile force, lb (N)
t = thickness of masonry element, in. (mm)
t_{fs} = concrete masonry unit face shell thickness, in. (mm)
V = applied shear force, lb (N)
V_r = shear capacity (resisting shear) of masonry, lb (N)
η = modular ratio
ρ = reinforcement ratio

REFERENCES

1. *Building Code Requirements for Masonry Structures*, ACI 530-02/ASCE 5-02/TMS 402-02. Reported by the Masonry Standards Joint Committee, 2002.
2. *Empirical Design of Concrete Masonry Walls*, TEK 14-8A. National Concrete Masonry Association, 2001.
3. *Strength Design of Concrete Masonry*, TEK 14-4A. National Concrete Masonry Association, 2002.
4. *Minimum Design Loads for Buildings and Other Structures*, ASCE 7-02. American Society of Civil Engineers, 2002.
5. *2000 International Building Code*. Falls Church, VA: International Code Council, 2003.
6. *Section Properties of Concrete Masonry Walls*, TEK 14-1. National Concrete Masonry Association, 2003.
7. *Steel Reinforcement for Concrete Masonry*, TEK 12-4C. National Concrete Masonry Association, 2002.
8. *Concrete Masonry Design Tables, TR 121*. National Concrete Masonry Association, 2000.
9. *Concrete Masonry Wall and Lintel Design Software*, CMS10. National Concrete Masonry Association, 2002.
10. *Loadbearing Concrete Masonry Wall Design*, TEK 14-5A. National Concrete Masonry Association, 2000.